Book of Abstracts

DelliRio 2025

Degenerate ellipticity in Rio de Janeiro

13th to 17th January, 2025.
Colégio Brasileiro de Altos Estudos.
Rio de Janeiro, Brazil.

Contributed Talks

1. Adina Ciomaga.

Homogenization of nonlocal Hamilton Jacobi equations.

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Abstract: I will present the framework of periodic homogenisation of nonlocal Hamilton-Jacobi equations, associated with Levy-Itô integro-differential operators. A typical equation is the fractional diffusion coupled with a transport term, where the diffusion is only weakly elliptical. Homogenization is established in two steps: (i) the resolution of a cellular problem - where Lipshitz regularity of the corrector plays a key role and (ii) the convergence of the oscillating solutions towards an averaged profile - where comparison principles are involved. I shall discuss recent results on the regularity of solutions and comparison principles for nonlocal equations, and the difficulties we face when compared with local PDEs.

The talked is based on recent developments obtained in collaboration with E. Topp, D. Ghilli, O. Ley, T.M. Le.

2. Alfredo Miranda.
Games for the two membranes problem.

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Abstract: We find viscosity solutions to the two membranes problem (that is, a system with two obstacle-type equations) with two different p-Laplacian operators taking limits of value functions of a sequence of games. We analyze two-player zero-sum games that are played in two boards with different rules in each board. At each turn both players (one inside each board) have the choice of playing without changing board or changing to the other board (and then playing one round of the other game). We show that the value functions corresponding to this kind of game converge uniformly to a viscosity solution of the two membranes problem. If in addition the possibility of having the choice to change boards depends on a coin toss we show that we also have convergence of the value functions to the two membranes problem that is supplemented with an extra condition inside the coincidence set.

3. Andrei Rodriguez-Paredes.
Large-time behavior of unbounded solutions of the viscous Hamilton-Jacobi equations.

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Abstract: We study the large-time behavior of solutions to the so-called *viscous Hamilton Jacobi equation*, $u_t - \Delta u + |Du|^m = f(x)$, over $\mathbb{R}^n$, with a particular focus on allowing as much generality as possible regarding growth at infinity for the source term f and the prescribed initial data. The subquadratic and superquadratic cases, $1 < m \leq 2$ and $m > 2$, respectively, are tretaed separately, in particular due to the lack of suitable comparison principles in the subquadratic case.

Joint works with Guy Barles and Alexander Quaas.

References:

Barles, G., Quaas, A., and Rodríguez-Paredes, A. *Large-time behavior of unbounded solutions of viscous Hamilton-Jacobi equations in* $\mathbb{R}^N$*.* Communications in Partial Differential Equations, 46(3):547–572, 2020.

Quaas, A., and Rodríguez-Paredes, A. *Large-time behavior of unbounded solutions of the viscous Hamilton–Jacobi equation: quadratic and subquadratic cases,* Israel Journal of Mathematics, https://doi.org/10.1007/s11856-024-2703-0, 2024.

4. Boyan Sirakov.

Elliptic regularity estimates with optimized constants and applications.

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Abstract: We revisit the classical theory of linear second-order uniformly elliptic equations in divergence form whose solutions have Hölder continuous gradients, and prove versions of the generalized maximum principle, the $C^{1,\alpha}$-estimate, the Hopf-Oleinik lemma, the boundary weak Harnack inequality and the differential Harnack inequality, in which the constant is optimized with respect to the norms of the coefficients of the operator and the size of the domain. Our estimates are complemented by counterexamples which show their optimality. We also give applications to the Landis conjecture and spectral estimates.

Joint work with Philippe Souplet (Université Sorbonne Paris Nord).

References:

Sirakov, B., and Souplet, P. *The Vázquez maximum principle and the Landis conjecture for elliptic PDE with unbounded coefficients*, Adv. Math. 387 (2021), Paper No. 107838, 27 pp.

Sirakov, B., and Souplet, P. *Elliptic regularity estimates with optimized constants and applications,* https://arxiv.org/abs/2411.19367

5. Damião Araújo.

Regularity in diffusion models with gradient activation.

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Abstract: In this talk, we discuss sharp regularity estimates for solutions of highly degenerate fully nonlinear elliptic equations. These are free boundary models in which a nonlinear diffusion process drives the system only in the region where the gradient surpasses a given threshold.

This is joint work with Aelson Sobral, King Abdullah University of Science and Technology, KAUST - Saudi Arabia, and Eduardo Teixeira, University of Central Florida - EUA.

6. Jean Carlos Nakasato.

Homogenization of the heat equation in a time-oscillatory moving thin domain.

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Abstract: We study the asymptotic behavior of solutions of the heat equation in a time oscillatory thin domain. We start by determining estimates of solutions in the perturbed domain and then find the effective problem. This is a joint work with Tatsu-Hiko Miura (University of Hirosaki).

7. João Vitor da Silva.

Regularity estimates for weighted quasilinear elliptic equations of p-Laplacian type.

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Abstract: In this Lecture, we obtain sharp and improved regularity estimates for weak solutions of weighted quasilinear elliptic models of Hardy-Hénon-type, featuring an explicit regularity exponent depending only on universal parameters. We also establish higher regularity estimates and non-degeneracy properties in some specific scenarios, providing further geometric insights into such solutions. Our regularity estimates both enhance and, to some extent, extend the results arising from the $C^{p'}$ conjecture for the p-Laplacian with a bounded source term. Finally, our results are noteworthy, even in the simplest model case governed by the p-Laplacian with regular coefficients:

$$\operatorname{div}\left(|\nabla u|^{p-2}\mathfrak{A}(|x|)\nabla u\right) = |x|^{\alpha}u_{+}^{m}(x) \quad \text{in} \quad B_1$$

under suitable assumptions on the data, with possibly singular weight $\mathfrak{h}(|x|) = |x|^{\alpha}$, which includes the Matukuma and Batt– Faltenbacher–Horst's equations as toy models.

References:

Araújo, D.J., Teixeira, E.V. and Urbano, J.M. *A proof of the $C^{p'}$-regularity conjecture in the plane.* Adv. Math. 316 (2017), 541-553.

Indrei, E., Minne, A. and Nurbekyan, L. *Regularity of solutions in semilinear elliptic theory.* Bull. Math. Sci. 7 (2017), no.1, 177-200.

Shahgholian, H. *$C^{1,1}$ regularity in semilinear elliptic problems.* Commun. Pure Appl. Math. 56 (2), 278-281 (2003).

Teixeira, E.V. *Regularity for quasilinear equations on degenerate singular sets.* Math. Ann. 358 (2014), no.1-2, 241-256.

Teixeira, E.V. *On the critical point regularity for degenerate diffusive equations.* Arch. Ration. Mech. Anal. 244 (2022), no.2, 293–316.

8. Jose Miguel Urbano.

Improved moduli of continuity for degenerate phase transitions.

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Abstract: We substantially improve in two scenarios the current state-of-the-art modulus of continuity for weak solutions to the N-dimensional, two-phase Stefan problem featuring a $p-$degenerate diffusion: for $p = N \geq 3$, we sharpen it to

$$\boldsymbol{\omega}(r) \approx \exp(-c|\ln r|^{\frac{1}{N}});$$

for $p > \max\{2, N\}$, we derive an unexpected Hölder modulus.

This is joint work with Ugo Gianazza and Naian Liao.

9. Juliana Fernandes.
On Bifurcation from Infinity: A Compactification Approach.

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Abstract: We will introduce a compactification method that has been successfully applied in various contexts, such as in non-dissipative reaction-diffusion equations, where it provides a clear framework for studying unbounded solutions, their stability, and related phenomena like heteroclinic orbits. After discussing the general application of this method in broader settings, we will focus on its specific use in the context of scalar parabolic partial differential equations with nonlinear boundary conditions. In this case, we will examine how the compactification technique aids in understanding bifurcations from infinity, the associated equilibria, and their connection to the Steklov eigenfunctions.

This is joint work with P. Lappicy and J. M. Arrieta.

10. Julio Hoyos.
The geometry of free boundaries in PDE.

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Abstract: Free boundary problems naturally arise in phenomena such as ice melting and iron behavior under abrupt temperature changes during forging (quenching). These problems can be formulated as seeking a pair (Ω, u), where Ω represents the domain and u is a function satisfying a specific partial differential equation (PDE) in Ω.

Understanding the domain's geometry is a crucial yet challenging aspect, with open questions dating back to the 1980s. This lecture will cover some classical results in the linear and nonlinear scenarios. Key differences between this scenarios will be discussed, highlighting the challenges of the nonlinear case and compare the methods in each case. This comparison unveils some geometric results in the nonlinear case.

11. Marcone Pereira.
Partial Differential Equations in Thin Domains

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Abstract: In this talk, we will explore results related to parabolic and elliptic equations defined in thin domains. We will begin by examining the pioneering work of Hale and Raugel (JDE, 1992) in the context of parabolic equations. From there, we will move to more recent developments where Homogenization Theory plays a crucial role in analyzing the asymptotic behavior of solutions as a domain in Euclidean space degenerates into an interval.

12. María Medina de la Torre.
Blowing-up solutions to critical competitive systems in dimensions 3 and 4.

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Abstract: We will analyze the existence and the structure of different sign-changing solutions to the Yamabe equation in the whole space and we will use them to find positive solutions to critical competitive systems in dimensions 3 and 4.

13. Martino Bardi.

Asymptotic properties of some non-coercive Hamiltonians.

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Abstract: Several interesting asymptotic properties of Hamilton-Jacobi equations are based on the so-called critical value of the Hamiltonian $H(x,p)$ and on the associated critical stationary H-J equation. In particular, the long time behaviour of evolutive H-J equations is described in terms of the critical value and a critical solution, and so is the homogenisation of H-J equations with highly oscillating ingredients. The theory was pioneered by Lions, Papanicolaou and Varadhan and by A. Fathi, and it has applications to ergodic control and to dynamical systems, the so-called weak KAM theory. Most of the known result assume the coercivity of the Hamiltonian in the moment variables p, and interpret the critical equation in the sense of continuous viscosity solutions. After reviewing some classical results, I'll present some recents improvements holding for non-coercive Hamiltonians arising from the optimal control of affine systems, possibly with an uncontrolled drift term. Different from the previous theory, I use in a crucial way viscosity solutions that can be discontinuous.

14. Olivier Ley.

Nonlocal Hamilton-Jacobi equations on networks with Kirchhoff conditions

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Abstract: This is a joint work with Guy Barles (Tours) and Erwin Topp (Rio) in which we study the well-posedness of nonlocal Hamilton-Jacobi on networks with Kirchhof conditions at the vertices. We are able to deal with integro-differential with order strictly less than one. I will explain how we define solutions of such equations on a network, how we construct them and how we extend the approach of Lions & Souganidis to prove uniqueness.

15. Paulo Amorim

Some uses of parabolic equations in mathematical biology: from predators to ants.

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Abstract: In this talk we will explore some applications of parabolic (and elliptic) equations in mathematical biology. We will first focus on their use in the modeling of spatial population dynamics involving pursuit and evasion behaviors, for instance in predator-prey models. This gives rise to systems of parabolic and parabolic-elliptic equations of chemotaxis type, of which we show some examples. Next, we show how parabolic equations can be used to describe aspects of ant behavior. The first example is an attempt to describe the trail-following behavior of ants through a system of parabolic equations. Its analysis shows the use of some interesting analytical techniques, and how this even gives rise to new estimates. Our next example is a coupled parabolic-ODE model, where the trajectories of individual ants are collectively and non-locally coupled through a parabolic equation for the pheromone they produce and follow. We show theoretical and numerical results. Finally, we show how parabolic equations can be used to model territory dynamics in ants, and are able to shed light on observations of conflict areas and "no-ants-land" observed in the field.

16. Wladimir Neves.

An analysis of the 2-D isentropic Euler Equations for a generalized polytropic gas law.

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Abstract: In this talk we develop an analysis of the compressible, isentropic Euler equations in two spatial dimensions for a generalized polytropic gas law. The main focus is rotational flows in the subsonic regimes, described through the framework of the Euler equations expressed in self-similar variables and pseudo-velocities. A Bernoulli type equation is derived, serving as a cornerstone for establishing a self-similar system tailored to rotational flows. In the final section, the study extends to an analysis of a perturbed model, introducing the concept of quasi-potential flows, offering insights into their behavior and implications.